INDICES

Summary:

1. *The expression* 3^5 *is read as* ***3*** *to power* ***5*** *and is defined as follows:*

$3^5 = 3 \times 3 \times 3 \times 3 \times 3$ ***("3"*** *is called the base and* ***"5"*** *is the index or power)*

2. *The following rules apply to indices:*

(i) $a^m \times a^n = a^{m+n}$

To multiply expressions with the same base, copy the base and add the powers.

(ii) $a^m \div a^n = a^{m-n}$

To divide expressions with the same base, copy the base and subtract the

powers.

(iii) $\left(a^m\right)^n = a^{mn}$

To raise an expression with the ***nth*** *power, copy the base and multiply the*

powers.

(iv) $a^0 = 1$

Any number to power zero is ***1*** *except zero.*

(v) $a^{-n} = \frac{1}{a^n}$

Any number raised with a negative power, is the reciprocal of the number with

a positive power.

$$\therefore \left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^{n}$$

(vi) $\sqrt{a} = a^{\frac{1}{2}}$

The square root of any number is that number raised to power a half.

(vii) $\sqrt[n]{a} = a^{\frac{1}{n}}$

The ***nth*** *root of any number is that number raised to power* $\frac{1}{n}$.

$$\therefore \left(\sqrt[n]{a}\right)^{m} = a^{\frac{m}{n}}$$

(viii) $a^m = a^n$ *implies that* $m = n$

(ix) $(a \times b)^m = a^m \times b^m$

EXAMPLES:

1. *Write the following expanded forms in index form:*

(i) $5 \times 5 \times 5 \times 5$

(ii) $3 \times 3 \times 3 \times 3 \times 3 \times 3$

(iii) $2 \times 2 \times 2 \times 2 \times 2$

(iv) $7 \times 7 \times 7$

2. *Write each of the following index forms in expanded form:*

(i) 10^3

(ii) 7^4

(iii) 9^5

3. *Write the following numbers in index form using base 2:*

(i) 8

(ii) 32

(iii) 16

(iv) 64

4. *Find the value of the following index forms:*

(i) 3^2

(ii) 2^3

(iii) 7^2

(iv) $(49)^{\frac{1}{2}}$

(v) $8^{\frac{2}{3}}$

(vi) $(125)^{\frac{1}{3}}$

(vii) $(0 \cdot 008)^{\frac{1}{3}}$

(viii) $\left(\sqrt[3]{125}\right)^2$

(ix) $\left(\frac{16}{81}\right)^{\frac{-3}{4}}$

5. Simplify the following:

(i) $y^7 \times y^3 \times y^5$

(ii) $y^{10} \div y^3$

(iii) $\frac{y^7 \times y^4}{y^5}$

(iv) $\frac{a^7 \times \left(ab^4\right)^2}{\left(a^2 \times b\right)^3}$

(v) $\frac{2^y \times 8^{y-1}}{16^{y-1}}$

(vi) $\frac{5^y \times 125^{y+1}}{625^{y+1}} + \frac{4^y \times 8^{y-2}}{32^{y-1}}$

6. Without using a calculator**,** simplify the following:

(i) $\frac{1}{2} - \left(\frac{25}{4}\right)^{\frac{-1}{2}}$

(ii) $(16)^{\frac{-1}{2}} + (32)^{\frac{-1}{5}} - \left(\frac{8}{125}\right)^{\frac{2}{3}}$

(iii) $4(0 \cdot 04)^{\frac{-1}{2}} - 8(4)^{-1}(16)^{\frac{3}{4}}$

(iv) $(27)^{\frac{-1}{3}}(25)^{\frac{1}{2}} - 5(0 \cdot 008)^{\frac{1}{3}}$

(v) $\left(\frac{1}{16}\right)^{\frac{-1}{2}} \times \left(\frac{1}{64}\right)^{\frac{-1}{3}}$

(vi) $\left(\frac{64}{81}\right)^{\frac{-1}{2}} \times \frac{1}{3}(32)^{\frac{3}{5}}$

(vii) $\left(\frac{8}{125}\right)^{\frac{-2}{3}} \times \left(\frac{5}{8^{1/2}}\right)^{-2}$

(viii) $\frac{2^{-2} \times 3^{-3}}{2^{-4} \times 3^{-6} \times 18}$

(ix) $\frac{(12)^{\frac{3}{2}} \times 9^{\frac{1}{4}}}{(36)^{\frac{1}{2}}}$

(x) $\frac{40 \times \sqrt{0 \cdot 36}}{(27)^{\frac{2}{3}} \div (81)^{\frac{3}{4}}}$

(xi) $\frac{(12)^{\frac{3}{2}} \times (16)^{\frac{1}{8}}}{(27)^{\frac{1}{6}} \times (18)^{\frac{1}{2}}}$

7. Solve for x and y in the following equations:

(i) $$\begin{aligned} 3^{x-y} &= 27 \\ 3^{x+y} &= 243 \end{aligned}$$

(ii) $$\begin{aligned} 2^{2x-3y} &= 16 \\ 5^{x-2y} &= 1 \end{aligned}$$

8. Solve for y in the following equations:

(i) $y^{\frac{2}{3}} = 4$

(ii) $4^{y} = 32$

(iii) $2^{2y} = \frac{1}{8}$

(iv) $\left(\frac{a}{b}\right)^{y-1} = \left(\frac{b}{a}\right)^{y-5}$

(v) $(32)^{\frac{3}{5}} \div y^{\frac{1}{2}} = 2$

(vi) $(32)^{y} \times \frac{1}{8} \times 4^{y+3} = 2^{24}$

(vii) $5^{3(y-1)} \times 125 = 625$

(viii) $\frac{243 \times 3^{2y}}{729 \times 3^{y} \div 3^{2y-1}} = 81$

(ix) $3^{y} \times 9^{\frac{-2}{y}} = 1$

(x) $27 \times 3^{2y} = \left(3^{y}\right)^{y}$

(vi) $(16)^{y^2} = 8^{4y-3}$

(vii) $4 \times 2^{3(y-1)} = 2^{-y}$

EER

1. *Simplify the following:*

(i) $\frac{3^{2n} \times 9^{n-1}}{81^{n-1}}$

(ii) $\frac{9^{n+1} \times 3^{n-1} \times 2^{n+1}}{27^{n-1} \times 2^{n}}$

(iii) $\frac{9^{n+1} \times 6^{n-1}}{3^{3n-1} \times 2^{n}}$

(iv) $\frac{243^{\frac{n}{5}} \times 3^{2n+1}}{9^{n} \times 3^{n-1}}$

2. *Without using a calculator, evaluate:* $\frac{3 \times 10^{4} \times \left(6 \times 10^{5}\right)^{2}}{90 \times 10^{3}}$

3. *Without using a calculator, simplify* $(27)^{\frac{-1}{3}}(25)^{\frac{1}{2}} - 5(0\cdot008)^{\frac{1}{3}}$

4. *Solve for* **x** *and* **y** *in the equation:* $\dfrac{3^x \times 2^{2y}}{3^y \times 2^x} = 72$

5. *Without using a calculator, simplify* $\left(\dfrac{81}{16}\right)^{\frac{-1}{4}} - \left(\dfrac{8}{27}\right)^{\frac{2}{3}}$

6. *If* $5^n = 3125$, *find the value of* $5^{(n-3)}$

7. *If* $81^x = 729^y$, *find the value of* $\dfrac{y}{x}$

8. *Without using a calculator, simplify* $\left(\dfrac{16}{81}\right)^{\frac{-3}{4}} \div \left(\dfrac{36}{64}\right)^{\frac{3}{2}}$

9. *Solve for* **x** *in the following equations:*

(i) $5^{7x-2} = 125^x$

(ii) $3^x = 9^{x+3}$

(iii) $81^x = \left(\dfrac{1}{3}\right)^{x-10}$

(vi) $4^{6-9x} = \dfrac{1}{8^{x-2}}$

10. *Solve for* **x** *in the following equations:*

(i) $5^{x^2} = 5^{6-x}$

(ii) $16^{x^2} = 8^{4x-3}$

(iii) $3^{x^2} \times 9^{-2} = 1$

(iv) $3^x \times \dfrac{1}{729} = 9^{x^2}$

11. *Solve for* **x** *and* **y** *in the equations:*

$$3^{x-y} = 9$$
$$2^{x+y} = 32$$

12. *If* $x = 32$, *find the value of* $\left(x^{\frac{1}{5}} \right)^{-1} + 3\left(\frac{x}{2} \right)^{\frac{-1}{2}}$

13. *Given that* $\frac{4^x \times 2^y}{2^{x+2y}} = 2^p$, *express* p *in terms of* x *and* y

14. *Given that* $3^{2x-y} = 27$ *and* $5^x \div 25^y = 1$, *solve for* x *and* y

15. *Given that* $a = -2$ *and* $b = 2$, *find the value of* $a^b - b^a$

16. *Without using a calculator or tables, evaluate:* $\sqrt{\frac{0{\cdot}24 \times 1{\cdot}56 \times 7{\cdot}2}{1{\cdot}3 \times 0{\cdot}16 \times 0{\cdot}09}}$

17. *Without using a calculator or tables, evaluate:* $\sqrt[3]{\frac{0{\cdot}064 \times 0{\cdot}125}{0{\cdot}008 \times 0{\cdot}001}}$

STANDARD FORM

Summary:

1. *Standard form is a way of expressing a number in the form* $A \times 10^n$ *where* $1 \leq A < 10$ *and* n *is an integer*

2. (i) *To express a number in standard form, we shift the decimal point until the digit part* A *is between* **1** *and* **10.** *This digit* A *has a decimal point placed after the first digit.*

(ii) *The power part* (10^n) *shows how many places to move the decimal point*

3. *The rules of indices apply to calculations in standard form*

EXAMPLES:

1. *Express the following numbers in standard form:*

(i) 25000 (ii) 3860 (iii) 568·3 (iv) 74·8 (v) 3·584 (vi) 4000

(vii) 0·435 (viii) 0·000263 (ix) 0·007 (x) 0·00356 (xi) 248×10^{3}

(xii) 4500×10^{-7} ***(xiii)*** $58 \cdot 4 \times 10^{-4}$ ***(xiv)*** $0 \cdot 027 \times 10^{-3}$

(xv) $0 \cdot 0062 \times 10^{-4}$ ***(xvi)*** $0 \cdot 0364 \times 10^{5}$

2. *By expressing each of the numbers in standard form, evaluate the following:*

(i) 0·0004 × 0·002 (ii) 0·005 × 0·00004 (iii) 800000 × 0·0005

(iv) $\frac{800}{0 \cdot 004}$ ***(v)*** $\frac{0 \cdot 00006}{2000}$ ***(vi)*** $\frac{0 \cdot 0009 \times 8000}{0 \cdot 002 \times 0 \cdot 3}$

3. *By expressing each of the numbers in the form* $a \times 10^{n}$ *where* ***n*** *is even, evaluate the following:*

(i) $\frac{0 \cdot 81}{0 \cdot 0027}$ ***(ii) 0·02 × 0·0015 (iii) 1500 × 40000***

4. *By expressing each of the numbers in the form* $a \times 10^{n}$ *where* ***n*** *is even, find the square root of:*

(i) 0·25 × 0·64 (ii) 0·16 × 0·09 (iii) 0·0036 × 0·25 (iv) 0·0081 × 0·0004

5. *Without using a calculator, evaluate the following give your answer in standard form:*

(i) $(2 \times 10^{2}) \times (6 \times 10^{5})$ ***(ii)*** $(4 \times 10^{-3}) \times (5 \times 10^{4})$

(iii) $(9 \times 10^{-2}) \times (4 \times 10^{-3})$ ***(iv)*** $(15 \times 10^{-3}) \times (20 \times 10^{7})$

6. *Without using a calculator or tables, evaluate:* $\frac{1{\cdot}21 \times 10^{-2} \times 40}{2{\cdot}2 \times 11}$

7. *Without using a calculator or tables, evaluate:* $\frac{2{\cdot}25 \times 10^{-3} \times 3{\cdot}9}{1{\cdot}3 \times 1{\cdot}5}$

EER:

1. *Without using a calculator, evaluate:* $\frac{0{\cdot}0035}{0{\cdot}07 \times 0{\cdot}2}$

2. *Without using a calculator, evaluate:* $\frac{0{\cdot}625 \times 0{\cdot}009}{0{\cdot}0045}$

2. *Without using a calculator, evaluate:* $\frac{0{\cdot}42 \times 0{\cdot}35 \times 0{\cdot}0015}{0{\cdot}049 \times 0{\cdot}003}$

3. *Without using a calculator, evaluate:* $\frac{1{\cdot}69 \times 10^{-2} \times 1{\cdot}25}{1{\cdot}3 \times 2{\cdot}5}$

4. *Without using a calculator, evaluate:* $\frac{5600}{80000}$ *give your answer in the form* $a \times 10^{n}$ *where* $1 \leq a < 10$ *and* n *is an integer*

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